Technical Note

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Motion of the Hexapod

Position and Orientation in Space, Center of Rotation

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Position and Orientation in Space

For simplification, the spatial position of the hexapod, i.e., the combination of its position and orientation in three-dimensional space is designated in this document as "pose".

The pose of a hexapod is defined by 6 coordinates:

Translation_X, Translation_Y, Translation_Z, Angle_U, Angle_V, Angle_W

A pose can be commanded with the following command:

MOV X Translation_X **Y** Translation_Y **Z** Translation_Z **U** Angle_U **V** Angle_V **W** Angle_W

The Translation_x, Translation_y, Translation_z coordinates designate a Cartesian move and the Angle_u, Angle_v, Angle_w coordinates designate a rotation in space.

In addition, the 6 coordinates for the pose of the hexapod refer to a center of rotation that is defined by the coordinates (P_R, P_S, P_T).

When the default settings for the coordinate system and the center of rotation are used, the center of rotation after a reference move is located at the origin of the coordinate system (0,0,0), see the dimensional drawing of the hexapod.

The center of rotation always moves together with the platform of the hexapod.

Depending on the enabled coordinate system, the center of rotation can be moved from the origin of the coordinate system in the X and/or Y and/or Z direction with the SPI command. The center of rotation that can be moved using the SPI command is also referred to as "pivot point".

The pivot point can be set with the following command

SPI R P_R **S** P_S **T** P_T

For further details on the center of rotation, see "The Role of the Center of Rotation when Defining the Pose" (p. 5).

In addition to the default settings for the coordinate system and center of rotation, users can define their own coordinate systems. To work with user-defined coordinate systems, see the C887T0007 Technical Note.

Definition of Rotation

The three angles Angle_U, Angle_V, Angle_W describe an orientation in space. The associated convention is designated "Cardan Matrix" or "Cardan Rotation".

Angle_U designates a rotation around the X axis, Angle_V a rotation around the Y axis, and Angle_W a rotation around the Z axis.

These elementary rotations are performed consecutively, whereby attention must be paid to the order of rotation.

Please note: In the following sections, the rotations are depicted mathematically (p. 3) and graphically (p. 4). Both forms of depiction differ in the underlying coordinate system: In one depiction, reference is made to a spatially fixed coordinate system, while the other depiction refers to a rotating coordinate system.

Mathematical Representation of the Rotation by a Rotation Matrix

The following describes how and in which order Angle_U, Angle_V, Angle_W are evaluated mathematically for a change in orientation:

The resulting rotation matrix is designated here by R.

$$Ur = \frac{Angle\_U\,\pi}{180}$$

$$Vr = \frac{Angle\_V\,\pi}{180}$$

$$Wr = \frac{Angle\_W\,\pi}{180}$$

$$\mathrm{Rx}(Ur) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(Ur) & -\sin(Ur) \\ 0 & \sin(Ur) & \cos(Ur) \end{bmatrix}$$

$$\mathrm{Ry}(Vr) := \begin{bmatrix} \cos(Vr) & 0 & \sin(Vr) \\ 0 & 1 & 0 \\ -\sin(Vr) & 0 & \cos(Vr) \end{bmatrix}$$

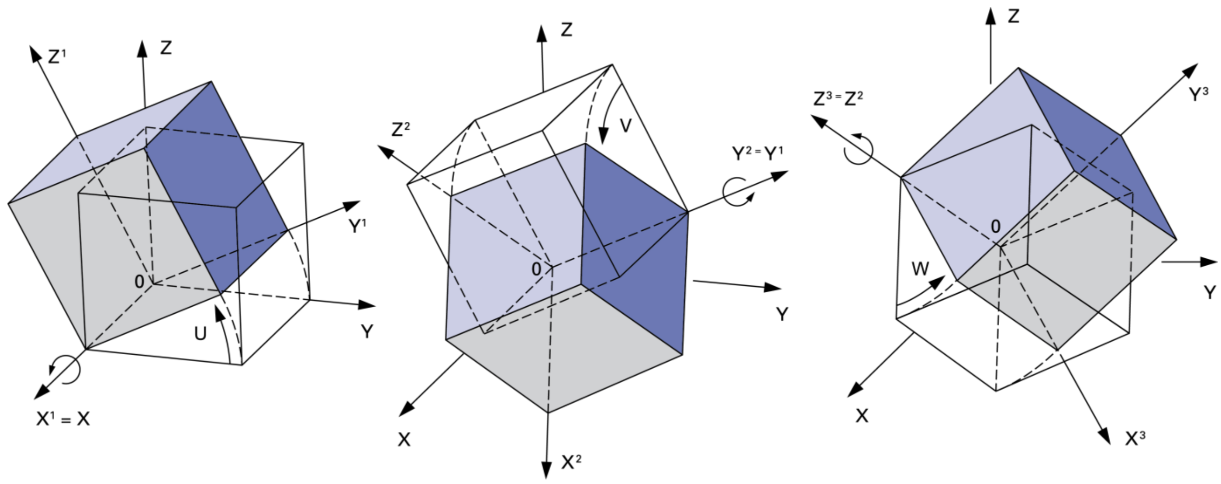
$$\mathrm{Rz}(Wr) := \begin{bmatrix} \cos(Wr) & -\sin(Wr) & 0 \\ \sin(Wr) & \cos(Wr) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{R}(Ur, Vr, Wr) := \mathrm{Rx}(Ur)\,.\,\mathrm{Ry}(Vr)\,.\,\mathrm{Rz}(Wr)$$

$$R = \begin{bmatrix} \cos(Vr)\cos(Wr) & -\cos(Vr)\sin(Wr) & \sin(Vr) \\ \cos(Ur)\sin(Wr)+\sin(Ur)\sin(Vr)\cos(Wr) & \cos(Ur)\cos(Wr)-\sin(Ur)\sin(Vr)\sin(Wr) & -\sin(Ur)\cos(Vr) \\ \sin(Ur)\sin(Wr)-\cos(Ur)\sin(Vr)\cos(Wr) & \cos(Ur)\sin(Vr)\sin(Wr)+\sin(Ur)\cos(Wr) & \cos(Ur)\cos(Vr) \end{bmatrix}$$

Graphical Representation of the Rotation

The following graphic visualizes the orientation that is assigned to the 3 coordinates Angle_U, Angle_V, Angle_W.



The order of the bodies depicted from left to right corresponds to the order of elementary rotations of the hexapod platform when reaching the pose:

1. Rotation around the X axis (left-hand body) at Angle_U.
2. Rotation around the transformed Y axis (middle body) at Angle_V
3. Rotation around the Z axis (right-hand body) at Angle_W.

Definition of the Pose by 6 Coordinates

Assume a pose of the hexapod with the following 6 coordinates:

Translation_X, Translation_Y, Translation_Z, Angle_U, Angle_V, Angle_W

When commanding motion, the position vector P1 (Px,Py,Pz) of the hexapod platform is transformed into position vector P2 by the rotation matrix R described above:

$P2^T = R*P1^T + (Translation\_X, Translation\_Y, Translation\_Z)^T$

In this representation, it is assumed that the center of rotation of the hexapod platform has the coordinates (0,0,0) in the initialization pose (normally after the reference move).

For motion with a center of rotation not equal to (0,0,0), see p. 6.

Order of Motion Commanding

At a given center of rotation, the pose of the hexapod platform is determined unequivocally by the 6 coordinates Translation_X, Translation_Y, Translation_Z, Angle_U, Angle_V, Angle_W.

All sequences of relative or absolute motion commands are equivalent to each other with respect to the resulting final pose when the 6 coordinates have the same values after the motion.

By commanding a part of the 6 coordinates, the values of exactly those coordinates are modified or redefined. The order of the elementary rotations is therefore independent of the commanding of individual coordinates.

The Role of the Center of Rotation When Defining the Pose

When translations in space are connected with changes of orientation in space, it is possible to describe this in several ways. In the case of hexapods, the center of rotation is used.

Definition of the center of rotation:

A center of rotation is a specific point of the rigid body to be moved. It does not need to be on the inside of this rigid body, but by its nature, it is a part of this rigid body. The center of rotation is also moved during all motion in space, which is described by the 6 coordinates.

In the figure above, the center of rotation is indicated by a red point.

The center of rotation (also: pivot point) of the hexapod platform can be moved by the SPI command; it then relates to a different locus of the hexapod platform.

A certain pose Translation_X, Translation_Y, Translation_Z, Angle_U, Angle_V, Angle_W can be reached in various different ways:

- Rotating of the hexapod platform with Angle_U, Angle_V, Angle_W around the center of rotation and following translation with Translation_X, Translation_Y, Translation_Z

or

- Translation of the hexapod platform with Translation_X, Translation_Y, Translation_Z and the following rotation with Angle_U, Angle_V, Angle_W around the center of rotation (also moved during translation)

Both rules lead to the same pose of the hexapod platform. It now becomes clear that the directions of translation are not just influenced by the rotations.

Motion with a Center of Rotation not Equal to (0,0,0)

If the coordinates for the center of rotation are changed from the values (0,0,0) to values not equal to (0,0,0), (e.g., with the SPI command), motion commanding with unchanged values for Translation_X, Translation_Y, Translation_Z, Angle_U, Angle_V, Angle_W generally leads to different poses.

A motion command with values for Translation_X, Translation_Y, Translation_Z, Angle_U, Angle_V, Angle_W that is evaluated together with a center of rotation P(Px,Py,Pz), leads to the following

$$P2^T = R*(P1^T - (Px,Py,Pz)^T) + (Px,Py,Pz)^T + (\text{Translation\_X},\text{Translation\_Y},\text{Translation\_Z})^T$$

The position vector P1 (Px,Py,Pz) of the hexapod platform is then transformed into a position vector P2.